AP Calculus BC Summer Assignment

These problems are essential practice for AP calculus BC. Unlike AP Calculus AB, BC students need to also be quite familiar with polar and parametric equations, as well as vectors and, series and sequences.

Please show all work either in the margins or on a separate sheet of paper. No credit will be given without supporting work.

If you have questions, please feel free to email me at satterwhitej@santarosa.k12.fl.us

This packet will be due on the first day of class.

Have a great Summer! See you in August!

Jim Satterwhite
AP Calculus AB and BC Teacher
1. Convert from parametric to rectangular:

\[ x = 3t \]
\[ y = 9t^2 \]

1. \( y = x \)
2. \( y = x^2 \)
3. \( y = x^3 \)
4. \( y = \sqrt{x} \)

2. Convert from parametric to rectangular:

\[ x = t \]
\[ y = \sqrt{t} \]

1. \( y = x \)
2. \( y = x^2 \)
3. \( y = x^3 \)
4. \( y = \sqrt{x} \)

3. Convert the following parametric equations into a rectangular equation.

\[ x = -2 \sin T \]
\[ y = 2 \cos T \]

1. \( x^2 - y^2 = 4 \)
2. \( x^2 + y^2 = 4 \)
3. \( y^2 = x + 4 \)
4. \( 4x^2 + 4y^2 = 4 \)

4. Convert the following parametric equations into a rectangular equation.

\[ x = -5 + \frac{T}{4} \]
\[ y = -1 + \frac{T}{4} \]

1. \( y = x + 4 \)
2. \( y = (x + 4)^2 \)
3. \( x - y = 4 \)
4. \( y^2 = x + 4 \)
5. Convert the following parametric equations into a rectangular equation.
   \[ x = \csc T \]
   \[ y = \cot T \]
   1. \( x^2 + y^2 = 1 \)
   2. \( y = \sqrt{x^2 - 1} \)
   3. \( x^2 - y^2 = 1 \)
   4. \( y^2 = \pm \sqrt{x^2 + 1} \)

6. \( x = 4\cos t \) and \( y = 2\sin t \) are equivalent to which of the following?
   1. linear function
   2. ellipse
   3. circle
   4. hyperbola

7. \( x = 4\cos t \) and \( y = 4\sin t \) is equivalent to which of the following?
   1. circle with radius 4
   2. circle with radius 2
   3. hyperbola
   4. ellipse with \( a = 2 \) and \( b = 4 \)

8. Which of the following best describes the graph of \( x = 5t, y = 3-3t, 0 \leq t \leq 1 \)?
   1. circle
   2. ellipse
   3. line segment
   4. line

9. What is the starting point of \( x = 5t, y = 3-3t, 0 \leq t \leq 1 \)?
   1. (-5,6)
   2. (0,-3)
   3. (0,3)
   4. (5,0)

10. What is the rectangular equation for the parameters \( x = \sin(2\pi t) \) and \( y = \cos(2\pi t) \) for \( 0 \leq t \leq 1 \)?
    1. \( x = y^2 \)
    2. \( y = x^2 \)
    3. \( x^2 + y^2 = 1 \)
    4. \( y = x + 2\pi \)

11. The graph of the equation \( \theta = \frac{\pi}{10} \) is a
    1. circle
    2. cardioid
    3. straight line
    4. lemniscate
12. Which polar equation is equivalent to the rectangular equation \( y = x^2 - x + 7 \)?

1. \( r^2 = r \cos^2 \theta - r \cos \theta + 7 \)
2. \( r \cos \theta = (r \sin \theta)^2 - r \sin \theta + 7 \)
3. \( r = \frac{(r \sin \theta)^2 - r \sin \theta + 7}{\cos \theta} \)
4. \( r = \sqrt{\left( \frac{r \cos \theta - \frac{1}{2} \right)^2 + \frac{27}{4}}}{\sin \theta} \)

13. Which rectangular equation is equivalent to the polar equation \( \sqrt{x^2 + y^2} = y \)?

1. \( r^2 = \sin \theta \)
2. \( r^4 = \sin \theta \)
3. \( r = \sin \theta \)
4. \( r^2 = \cos^4 \theta \)

14. Convert the rectangular equation \( x^2 + (y + 3)^2 = 13 \) into polar form.

1. \( r^2 + r \sin \theta = -2 \)
2. \( r = \sqrt{6} \sin \theta - 2 \)
3. \( r^2 + 6r \sin \theta = 4 \)
4. \( r^2 = 6r \sin \theta + 4 \)

15. Convert the following rectangular equation into polar form:

\[ \left( x^2 + y^2 \right)^2 = x^2 \]

1. \( r^2 = \cos \theta \)
2. \( r^2 = r \cos \theta \)
3. \( r = \cos \theta \)
4. \( r^2 = \cos^4 \theta \)

16. Convert the rectangular equation \( x^2 + y^2 = 6 \) into polar form.

1. \( r^2 = 6 \)
2. \( r = 6 \sin \theta \)
3. \( r = \sqrt{6} \)
4. \( r \cos \theta = 6 \)
17. Convert \( p = \frac{-4 \tan \theta}{\sin \theta} \) to a rectangular equation.
   1. \( x = 4 \)
   2. \( x = -4 \)
   3. \( y = -4 \)
   4. \( y^2 = 4 \)

18. Convert \( r = 5 \) to a rectangular equation.
   1. \( x = 5 \)
   2. \( y^2 = x^2 + 25 \)
   3. \( x^2 + y^2 = 25 \)
   4. \( x^2 = 25 \)

19. A vector \( \overrightarrow{RW} = (9, -5) \) has \( R(3, 4) \) as its initial point. What are the coordinates of the terminal point \( W \)?
   1. \((6, -7)\)
   2. \((-6, -9)\)
   3. \((12, -1)\)
   4. \((-6, 9)\)

20. Find the magnitude and the direction angle, in radians, of the vector \( <2, 2> \)
   1. \(8, \pi/3\)
   2. \(\sqrt{8}, \pi/4\)
   3. \(4, \pi/6\)
   4. \(4, \pi/4\)

21. Find the magnitude and the direction angle, in radians, of the vector \( <-\sqrt{2}, \sqrt{2}> \)
   1. \(2, 3\pi/4\)
   2. \(2, 5\pi/4\)
   3. \(4, \pi/4\)
   4. \(4, 7\pi/4\)

22. What is the magnitude and angle of direction, in radians, of the vector \( <\sqrt{3}, 1> \)
   1. \(1 + \sqrt{3}, \pi/3\)
   2. \(2, \pi/4\)
   3. \(1 + \sqrt{3}, \pi/6\)
   4. \(2, \pi/6\)

23. Find the magnitude and direction, in radians, of the vector \( <-5, 0> \)
   1. \(5, \pi\)
   2. \(25, \pi\)
   3. \(5, \pi/2\)
   4. \(25, 2\pi\)
24. Find the magnitude and direction, in radians, of the vector $<0, 4>$
   1. $16, \pi/2$
   2. $4, \pi/2$
   3. $16, \pi$
   4. $4, -\pi/2$

25. What is the value of $\sum_{x=0}^{2}(3-2a)^x$?
   1. $4a^2 - 2a + 12$
   2. $4a^2 - 2a + 13$
   3. $4a^2 - 14a + 12$
   4. $4a^2 - 14a + 13$

26. Select the common ratio and the 4th term of the geometric series: 9, −6, 4…
   1. Common Ratio: $-\frac{3}{2}$; 4th Term: $-13.5$
   2. Common Ratio: $-\frac{2}{3}$; 4th Term: $-2 \frac{2}{3}$
   3. Common Ratio: $-\frac{2}{3}$; 4th Term: $2 \frac{2}{3}$
   4. Common Ratio: $\frac{2}{3}$; 4th Term: $2 \frac{2}{3}$

27. The 4th term in a geometric sequence is −15, the 5th term is 45, and the 6th term is −135. Find the sum of the first 7 terms.
   1. $-105$
   2. $-101 \frac{1}{9}$
   3. $303 \frac{8}{9}$
   4. $305$

28. The first term in a geometric sequence is $\frac{1}{9}$ and the common ratio is −3. Find the 7th term in the sequence.
   1. $-243$
   2. $-\frac{1}{2187}$
   3. $\frac{1}{729}$
   4. $81$
30. The common ratio of the sequence $\frac{-1}{2}, \frac{5}{4}, \text{ and } -\frac{9}{8}$ is

1. $\frac{-3}{2}$
2. $-\frac{2}{3}$
3. $-\frac{1}{2}$
4. $-\frac{1}{4}$

31. Which expression is equivalent to the sum of the sequence 6, 12, 20, 30?

1. $\sum_{n=1}^{\infty} 2^n - 10$
2. $\sum_{n=1}^{\infty} \frac{2n^2}{3}$
3. $\sum_{n=1}^{\infty} 5n - 4$
4. $\sum_{n=1}^{\infty} n^2 + n$

32. A jogger ran $\frac{1}{3}$ mile on day 1, and $\frac{2}{3}$ mile on day 2, and $1 \frac{1}{3}$ miles on day 3, and $2 \frac{2}{3}$ miles on day 4, and this pattern continued for 3 more days. Which expression represents the total distance the jogger ran?

1. $\sum_{n=1}^{3} \frac{1}{3}(2)^{n-1}$
2. $\sum_{n=1}^{3} \frac{1}{3}(2)^n$
3. $\sum_{n=1}^{3} 2 \left(\frac{1}{3}\right)^{n-1}$
4. $\sum_{n=1}^{3} 2 \left(\frac{1}{3}\right)^n$
33. What is the common ratio of the sequence \( \frac{1}{64}a^5b^3, \frac{5}{32}a^3b^4, \frac{5}{16}ab^5 \)?
1. \( -\frac{3b}{2a^2} \)
2. \( -\frac{6b}{a^2} \)
3. \( -\frac{3a^2}{b} \)
4. \( -\frac{6a^2}{b} \)

34. The sum of the first eight terms of the series \( 3 - 12 + 48 - 192 + \ldots \) is
1. \(-13,107\)  
2. \(-21,845\)  
3. \(-39,321\)  
4. \(-65,535\)

35. What is the common ratio of the geometric sequence shown below?

\(-2, 4, -8, 16, \ldots\)

1. \( -\frac{1}{2} \)
2. \( 2 \)
3. \( -2 \)
4. \( -6 \)

36. Which summation represents \( 5 + 7 + 9 + 11 \ldots + 43 \)?

1. \( \sum_{n=5}^{48} n \)
2. \( \sum_{n=1}^{20} (2n+3) \)
3. \( \sum_{n=4}^{14} (2n-5) \)
4. \( \sum_{n=3}^{18} (3n-4) \)

37. What is the sum of the first 19 terms of the sequence 3, 10, 17, 24, 31,
1. 1188  
2. 1197  
3. 1254  
4. 1292

38. What is the fifteenth term of the sequence 5, -10, 20, -40, 80, \ldots?

1. \(-163,840\)  
2. \(-81,920\)  
3. 381,920  
4. 327,680
39. Find the sum of the series and tell if it converges or diverges.

\[-3 + 0.4 + 0.08 + 0.016 + 0.0032 + \ldots\]

1. sum = \(-3\), series diverges
2. sum = \(-\frac{5}{2}\), series converges
3. sum = \(\frac{1}{2}\), series converges
4. sum does not exist, series diverges

40. Find the sum of the series and tell if it converges or diverges.

\[0.3 + 0.6 + 1.2 + 2.4 + 4.8 + \ldots\]

1. sum = \(10\), series diverges
2. sum = \(10\), series converges
3. sum = \(2\), series converges
4. sum does not exist, series diverges

41. Find the sum of the series and tell if it converges or diverges.

\[\sum_{k=1}^{\infty} 3 \left(\frac{3}{2}\right)^k\]

1. sum = \(3\), series diverges
2. sum = \(3\), series converges
3. sum = \(0\), series converges
4. sum does not exist, series diverges

42. Find the sum of the series and tell if it converges or diverges.

\[\sum_{k=1}^{\infty} 2 \left(\frac{1}{5}\right)^k\]

1. sum = \(0.5\), series diverges
2. sum = \(0.5\), series converges
3. sum = \(0\), series converges
4. sum does not exist, series diverges
43. What are the first five terms of the sequence \( a_n = \frac{n+1}{n^2 - 6} \), and does the sequence converge or diverge?

1. \( \frac{2}{5}, \frac{3}{2}, \frac{4}{3}, \frac{1}{2}, \frac{6}{19} \) and the sequence diverges
2. \( \frac{2}{5}, \frac{3}{2}, \frac{4}{3}, \frac{1}{2}, \frac{6}{19} \) and the sequence converges
3. \( \frac{2}{5}, \frac{3}{2}, \frac{4}{3}, \frac{1}{2}, \frac{6}{19} \) and the sequence converges
4. \( \frac{-1}{6}, \frac{-2}{5}, \frac{-3}{2}, \frac{1}{3}, \frac{2}{2} \) and the sequence converges

44. Find the limit for the given sequence as ‘\( n \)’ increases without bound and tell if the sequence converges or diverges.

\[ a_n = \frac{3n - 2}{n} \]

1. Limit = 3, sequence converges
2. Limit = 3, sequence diverges
3. Limit does not exist, sequence converges
4. Limit does not exist, sequence diverges

45. Find the limit for the given sequence as ‘\( n \)’ increases without bound and tell if the sequence converges or diverges.

\[ a_n = \frac{6n + 8}{n^2} \]

1. Limit = 0, sequence diverges
2. Limit = 0, sequence converges
3. Limit does not exist, sequence converges
4. Limit does not exist, sequence diverges

46. Find the limit for the given sequence as ‘\( n \)’ increases without bound and tell if the sequence converges or diverges.

\[ a_n = \frac{1}{n^2 + 1} \]

1. Limit = 0, sequence diverges
2. Limit = 0, sequence converges
3. Limit does not exist, sequence converges
4. Limit does not exist, sequence diverges
47. The sum of a geometric series is \( \frac{55}{64} \) and its first term is \( \frac{5}{8} \). Find the common ratio of the series

1. \( -\frac{3}{11} \)
2. \( \frac{3}{11} \)
3. \( 0.33\overline{3} \)
4. \( 0.2727 \)

48. Find the common ratio of a geometric series if its sum is equal to ‘0.9’ and the first term is ‘0.5’

1. \( 0.44 \)
2. \( \frac{4}{9} \)
3. \( -\frac{4}{9} \)
4. \( 4.444444 \)

49. Express the series using sigma notation.

\[-5 + -7 + -9 + -11 + -13 + \ldots\]

1. \( \sum_{k=1}^{5} -2k - 3 \)
2. \( \sum_{k=2}^{5} 2^{k-1} - 3 \)
3. \( \sum_{k=0}^{x} -(3k + 2) \)
4. \( \sum_{k=1}^{x} -2k - 3 \)
50. Express the given series using sigma notation.

\[ 2 + 5 + 10 + 17 + 26 \]

1. \[ \sum_{k=1}^{26} k^2 + 1 \]
2. \[ \sum_{k=1}^{5} k^2 + 1 \]
3. \[ \sum_{k=1}^{5} (2k + 1)^2 \]
4. \[ \sum_{k=2}^{26} k^2 + 1 \]

51. In an arithmetic sequence where \( a_4 = -2 \) and \( a_{12} = 7 \), what is the explicit formula for the general term \( a_n \)?

1. \[ a_n = \frac{9}{8} n - \frac{13}{2} \]
2. \[ a_n = \frac{13}{2} n - \frac{9}{8} \]
3. \[ a_n = \frac{9}{8} n + \frac{13}{2} \]
4. \[ a_n = \frac{8}{9} n - \frac{2}{13} \]

52. In an arithmetic sequence where \( a_6 = 13 \) and \( a_{10} = 25 \), what is the explicit formula for the general term \( a_n \)?

1. \( a_n = 3n + 5 \)
2. \( a_n = 3n - 5 \)
3. \( a_n = -3n + 5 \)
4. \( a_n = 2n - 5 \)

53. Find the explicit formula for the following sequence:

\[ 10, 1, 0.1, 0.01, 0.001 \]

1. \[ a_n = 10(0.1)^{n-1} \]
2. \[ a_n = 10(0.1)^n \]
3. \[ a_n = 10(0.1)^n + 1 \]
4. \[ a_n = 100(0.1)^{n-1} \]
54. Find the explicit formula for the sequence:

10, 2, -6, -14

1. \( a_n = 8n + 18 \)
2. \( a_n = n + 26 \)
3. \( a_n = -8n + 18 \)
4. \( a_n = -8(n + 18) \)

56. Find the explicit formula for the following sequence.

\[
\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}
\]

1. \( a_n = \frac{1}{4^n} \)
2. \( a_n = \frac{1}{4^n} \)
3. \( a_n = \frac{1}{4^{4n}} \)
4. \( a_n = 4^{n-1} \)

57. Find the explicit formula for the following sequence.

1, 3, 5, 7, 9

1. \( a_n = 2n - 1 \)
2. \( a_n = n + 2 \)
3. \( a_n = 2n + 1 \)
4. \( a_n = 2n + 2 \)
58. What is the common ratio, \( r \), for the geometric sequence represented by the formula \( a_n = \left( \frac{2}{3} \right)^{n-1} \)?

1. \( \frac{1}{3} \)
2. \( \frac{2}{3} \)
3. 3
4. \( \frac{3}{2} \)

59. What is the common ratio, \( r \), for the following sequence?

\[ 28, 7, \frac{7}{4}, \frac{7}{16}, \ldots \]

1. \( \frac{1}{16} \)
2. 7
3. \( \frac{1}{4} \)
4. 4

60. Evaluate: \( \sum_{k=1}^{4} (k + 2)^3 \)

1. 216
2. 324
3. 432
4. 553

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Evaluate. Show supporting work for each problem (algebraic steps or sketch). No calculator.

62. \( \lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} \)
63. \( \lim_{x \to 0} \frac{(x - 5)^2 - 25}{x} \)
64. \( \lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} \)

65. \( \lim_{x \to -6} \frac{x + 6}{x^2 + 3x - 18} \)
66. \( \lim_{x \to -2} \frac{x^3 + 8}{x + 2} \)
67. \( \lim_{x \to \infty} \frac{3x - 5x^2}{4x^2 + 1} \)
Evaluate. Show supporting work for each problem (algebraic steps or sketch). No calculator.

68. \( \lim_{x \to 3} \frac{1}{x - 3} \)  
69. \( \lim_{x \to 3} \frac{1}{x - 3} \)  
70. \( \lim_{x \to 3} \frac{1}{x - 3} \)

71. \( \lim_{x \to 3} \frac{1}{(x - 3)^2} \)  
72. \( \lim_{x \to 3} |x - 1| \)  
73. \( \lim_{x \to 3} |x - 1| \)

74. \( f(x) = \begin{cases} 1 - x, & x \leq 1 \\ x^2, & x > 1 \end{cases} \) 
(a) \( \lim_{x \to 1^-} f(x) \)  
(b) \( \lim_{x \to 1^+} f(x) \)  
(c) \( \lim_{x \to 1} f(x) \)

75. \( f(x) = \begin{cases} x^2 - x - 6, & x \neq 3 \\ 4, & x = 3 \end{cases} \) 
(a) \( \lim_{x \to 3} f(x) \)  
(b) \( f(3) \)

Use the definition of the derivative to find the derivative. No calculator.

\[
\frac{f'(x)}{h} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}. \text{ (You must know this formula.)}
\]

76. \( f(x) = x^2 - 8x \)  
77. \( f(x) = \sqrt{x} + 9 \)

78. \( f(x) = \frac{3}{x - 4} \)  
79. \( f(x) = x^3 + 2x^2 - x + 4 \)

Use the differentiation rules (power rule, product rule, quotient rule) to find the derivative. Do not leave negative exponents or complex fractions in your answers. No calculator.

80. \( f(x) = 3x^4 - 5x^3 + \frac{2}{x} + 6x^{\frac{2}{3}} - 12 \)  
81. \( f(x) = \frac{2x^2 - 3x + 1}{x} \)

82. \( f(x) = \sqrt{x} + \sqrt[3]{x} \)  
83. \( f(x) = (6x + 5)(x^3 - 2) \)

84. \( f(x) = \frac{x^3 + 5x - 3}{x^2 - 1} \)

85. Given the function \( f(x) = x^4 - 3x^2 + 7 \).

(a) Use the differentiation rules to find \( f'(x) \).

(b) Write the equation of the tangent line to \( f \) at \((1, 5)\). Leave your equation in point-slope form.